[4] T. S. Chen, "Calculations of the parameters of ridge waveguides," *IRE Trans. Microwave Theory Tech.*, vol. MTT-5, pp. 12–17, Jan. 1957.
[5] J. R. Pyle, "The cutoff wavelength of the TE_{10} mode in ridged rectangular waveguide of any aspect ratio," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp. 175–183, Apr. 1966.
[6] J. P. Montgomery, "On the complete eigenvalue solution of ridged waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 547–555, June 1971.
[7] W. C. Young, Jr., "TE-mode solutions for dielectric slab center-loaded ridged waveguide," NRL Rep. 8105, Apr. 29, 1977.
[8] G. Magerl, "Ridged waveguides with inhomogeneous dielectric slab loading," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 413–416, June 1978.
[9] C. W. Young and G. Magerl, "Comments on ridged waveguides with inhomogeneous dielectric slab loading," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 919, Nov. 1978.
[10] R. F. Harrington, *Time-Harmonic Electromagnetic Fields.* New York: McGraw-Hill, 1961, pp. 381–388.
[11] S. W. Lee, W. R. Jones, and J. J. Campbell, "Convergence of numerical solutions of iris-type discontinuity problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 528–536, June 1971.

Alfred T. Villeneuve (S'52–A'53–M'58–SM'82) was born in Syracuse, NY. He received the B.E.E. degree from Manhattan College, Bronx, NY, in 1952, and the M.E.E. and Ph.D. degrees from Syracuse University, Syracuse, NY, in 1955 and 1959, respectively.

From 1952 to 1955, he was a Research Associate in Electrical Engineering at Syracuse University, where he worked on UHF antennas and microwave filters. From 1956 to 1959, he was an Instructor in Electrical Engineering, teaching courses in electromagnetism and network theory. In 1959, he was promoted to Assistant Professor. In the same year, he joined Hughes Aircraft Co., where he is currently a Senior Scientist in the Antenna Department of the Radar Systems Group. He has been engaged in various aspects of antenna theory and design there over the past 25 years. He has been a lecturer at the University of Southern California, the University of California at Los Angeles, and at Loyola-Marymount University in Los Angeles.

Dr. Villeneuve has served as a reviewer for the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES AND IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION and has served a term on the Administrative Committee of the Antennas and Propagation Society (AP-S). He also served as Secretary-Treasurer, Vice Chairman, and Chairman of the Los Angeles Chapter of AP-S. He is a member of the IEEE Antenna Standards Committee and was a member of Subcommittee 2.11 on Methods of Testing Antennas which recently published IEEE Std 149-1979, *IEEE Standard Test Procedures* for *Antennas*. He was a member of the Working Group on Definition of Terms for Antennas which recently completed IEEE Std 145-1983, *IEEE Standard Definitions of Terms for Antennas*. He served on the Steering Committees of the 1971 and 1981 International Antennas and Propagation Symposia and has also been on the Technical Program committees of these and several other symposia. He is a member of Eta Kappa Nu, Sigma Xi, and Commission B of the International Union of Radio Science.

Stability of Multifrequency Negative-Resistance Oscillators

BEVAN D. BATES, MEMBER, IEEE, AND PETER J. KHAN, SENIOR MEMBER, IEEE

Abstract —**A general criterion is derived for the stability of a negative-resistance oscillator with respect to small perturbations in the operating point. The derivation applies when the oscillator output consists of an arbitrary number of related frequency components, including possible nonharmonic components. Examples are given of the application of the stability criterion to coaxial IMPATT oscillator circuits, with experimental verification of the frequency and output power at theoretically determined stable operating points.**

I. Introduction

NEGATIVE-RESISTANCE devices find widespread application in microwave oscillators. As a consequence of the nonlinearity of the negative resistance and of the complicated frequency dependence of the impedance characteristic of the passive microwave circuit to which the device is connected, the resulting signal will generally contain harmonic components of the fundamental oscillation frequency. However, in the more general case, the frequency components in the oscillation may not be harmonically related due to parametric effects, and its up-converted low-frequency oscillation.

This paper presents expressions which permit determination of the stability of the oscillation state for the case where the device impedance is a function of both excitation and frequency, and an arbitrary number of frequency components are present. Use of the stability criteria derived here provides a more accurate determination of the oscillation characteristics of IMPATT and transferred-electron-device circuits using a realistic circuit model of the microwave mounting and impedance-transforming structure.

The oscillator stability studies derive from the fundamental work of Kurokawa [1], who developed a first-order theory describing the behavior of a one-port negative resistance embedded in a general passive multiple-resonant

Manuscript received October 24, 1983; revised May 17, 1984. Part of this work was performed at the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, and was supported by the National Research Council under an agreement with the National Aeronautics and Space Administration.
B. D. Bates was a NRC-NASA Resident Research Associate, Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, CA 91109. He is now with the Department of Electrical and Electronic Engineering, University of Melbourne, Victoria 3052, Australia.
P. J. Khan was with the Electrical Engineering Department, University of Queensland, St. Lucia, Queensland 4067, Australia. He is now with the Bahá'í World Centre, P.O. Box 155, Haifa, 31-001, Israel.

0018-9480/84/1000-1310$01.00 ©1984 IEEE

circuit: this led to a set of equations, having simple graphical interpretation, for a device with a frequency-independent impedance connected to a general linear network with a frequency-dependent impedance.

The restriction in the Kurokawa theory to a sinusoidal oscillator was relaxed by Brackett [2], who extended the theory to include a second-harmonic component. However, Brackett incompletely accounted for the harmonic relationship between the two frequencies, resulting in an incorrect formulation of the stability equations. Brackett also assumed that admittances y_{12} and y_{21}, which express frequency conversion between the two frequencies present, are both proportional to the fundamental voltage V_1 and independent of the harmonic voltage V_2. He also retained the assumption of frequency-independent device impedance. Foulds and Sebastian [3] applied the describing-function approach of Gustafsson *et al.* [4] to the study of oscillators with a second-harmonic voltage component present in addition to the fundamental. However, their stability analysis also failed to correctly account for the interaction between the fundamental and second-harmonic components. They consequently arrived at the incorrect conclusion that there exists a stability condition that must be satisfied at each harmonic frequency of interest. The analysis presented here, however, shows that, for purely harmonic interactions, only one stability condition must be satisfied regardless of the number of harmonics considered. Accurate determination of this condition, however, requires consideration of all the frequencies present in the system.

The present paper draws on the approaches of Brackett and of Foulds and Sebastian, but provides an analysis which is of greater generality and avoids the deficiencies of both these approaches.

II. Two-Frequency Oscillator Stability

To aid understanding and comparison with previous work, the analysis is set out initially for two-frequency components, i.e., the fundamental and the second harmonic, in the oscillator signal. In the next section, this analysis is generalized to an arbitrary number of frequency components which are not restricted to a harmonic relationship.

The derivation here is in terms of admittances, but could equally well be carried out with impedances. The analysis does not require the assumptions of proportional coupling and frequency-independent device impedances made by Brackett [2].

The time-varying voltage $V(t)$ across the device (Fig. 1), or the nonlinear portion of the device if its linear components are included in the coupling circuit, is given by

$$V(t) = V_1\cos(\omega_1 t + \phi_1) + V_2\cos(\omega_2 t + \phi_2)$$

with the assumption of only two-frequency components present and $\omega_2 = 2\omega_1$.

Application of the Kurokawa condition at the fundamental and second-harmonic frequencies gives

$$Y_{C_1}(\omega_1) + Y_{D_1}(\omega_1, V_1, V_2, \phi_1, \phi_2) = 0 \quad (1)$$

$$Y_{C_2}(\omega_2) + Y_{D_2}(\omega_2, V_1, V_2, \phi_1, \phi_2) = 0 \quad (2)$$

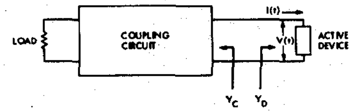

Fig. 1. General representation of multifrequency oscillator circuit, showing diode admittance Y_D and circuit admittance Y_C.

where

Y_{C_1}, Y_{C_2} are circuit admittances, and

Y_{D_1}, Y_{D_2} are diode admittances, with

the subscripts 1 and 2 denoting the fundamental and second-harmonic components, respectively. Equations (1) and (2) may be expressed in the form

$$p_k + jq_k = 0 \quad (3)$$

where

$$p_k = \mathrm{Re}(Y_{C_k} + Y_{D_k}) \text{ and } q_k = \mathrm{Im}(Y_{C_k} + Y_{D_k}), \qquad \text{for } k = 1, 2.$$

Consider small perturbations dp_k and dq_k in the operating point such that $p_k = p_{ko}$ under steady-state conditions and $p_k = p_{ko} + dp_k$ when the state is perturbed, and similarly for q_k. This is denoted by the notation

$$p_k \to p_{ko} + dp_k \quad (4)$$

and

$$q_k \to q_{ko} + dq_k. \quad (5)$$

Let the corresponding perturbations in the operating conditions be δV_k, $\delta\phi_k$, and $\delta\omega_k$ such that

$$V_k \to V_{ko} + \delta V_k \quad (6)$$

$$\phi_k \to \phi_{ko} + \delta\phi_k \quad (7)$$

$$\omega_k \to \omega_{ko} + \delta\omega_k. \quad (8)$$

Because δV_k, $\delta\phi_k$, and $\delta\omega_k$ are small quantities, (3) can be expanded in a Taylor series about the operating point. This gives the result

$$\begin{aligned}(p_{ko} + dp_k) + j(q_{ko} + dq_k) &= \left\{p_{ko} + \frac{\partial p_{ko}}{\partial V_1}\cdot\delta V_1 + \frac{\partial p_{ko}}{\partial V_2}\cdot\delta V_2 + \frac{\partial p_{ko}}{\partial \phi_1}\cdot\delta\phi_1 + \frac{\partial p_{ko}}{\partial \phi_2}\cdot\delta\phi_2 + \frac{\partial p_{ko}}{\partial \omega_1}\cdot\delta\omega_1 + \frac{\partial p_{ko}}{\partial \omega_2}\cdot\delta\omega_2\right\} \\ &+ j\left\{q_{ko} + \frac{\partial q_{ko}}{\partial V_1}\cdot\delta V_1 + \frac{\partial q_{ko}}{\partial V_2}\cdot\delta V_2 + \frac{\partial q_{ko}}{\partial \phi_1}\cdot\delta\phi_1 + \frac{\partial q_{ko}}{\partial \phi_2}\cdot\delta\phi_2 + \frac{\partial q_{ko}}{\partial \omega_1}\cdot\delta\omega_1 + \frac{\partial q_{ko}}{\partial \omega_2}\cdot\delta\omega_2\right\} = 0. \quad (9)\end{aligned}$$

Note that (3) is satisfied at the perturbed operating point only by a complex $\delta\omega_k$. Kurokawa [1] showed that, for small perturbations, δV_k and $\delta\phi_k$ are related by

$$\delta\omega_k = \frac{d(\delta\phi_k)}{dt} - \frac{j}{V_{ko}}\cdot\frac{d(\delta V_k)}{dt} \quad (10)$$

with δV_k and $\delta\phi_k$ assumed to be slowly varying functions of time t. Because $\omega_2 = 2\omega_1$, then $\delta\omega_2 = 2\delta\omega_1$, i.e.,

$$\frac{d(\delta\phi_2)}{dt} - \frac{j}{V_{20}} \cdot \frac{d(\delta V_2)}{dt} = 2\left[\frac{d(\delta\phi_1)}{dt} - \frac{j}{V_{10}}\frac{d(\delta V_1)}{dt}\right]. \tag{11}$$

Equating real and imaginary parts and integrating with respect to time, we obtain the relations

$$\delta\phi_2 = 2\delta\phi_1 + a \tag{12}$$

$$\frac{\delta V_2}{V_{20}} = \frac{2\delta V_1}{V_{10}} + b \tag{13}$$

where a and b are independent of time.

Substituting these values into (9) and using (10), we obtain

$$\begin{aligned}&\left\{\frac{\partial p_{ko}}{\partial v_1} \cdot \delta v_1 + \frac{\partial p_{ko}}{\partial v_2}(2\delta v_1 + b)\right.\\ &+ \frac{\partial p_{ko}}{\partial \phi_1} \cdot \delta\phi_1 + \frac{\partial p_{ko}}{\partial \phi_2}(2\delta\phi_1 + a)\\ &\left.+\left(\frac{\partial p_{ko}}{\partial \omega_1} + \frac{2\partial p_{ko}}{\partial \omega_2}\right)\cdot\left(\frac{d(\delta\phi_1)}{dt} - j\frac{d(\delta v_1)}{dt}\right)\right\}\\ &+ j\left\{\frac{\partial q_{ko}}{\partial v_1} \cdot \delta v_1 + \frac{\partial q_{ko}}{\partial v_2}(2\delta v_1 + b) + \frac{\partial q_{ko}}{\partial \phi_2} \cdot \delta\phi_1\right.\\ &+ \frac{\partial q_{ko}}{\partial \phi_2}(2\delta\phi_1 + a) + \left(\frac{\partial q_{ko}}{\partial \omega_1} + \frac{2\partial q_{ko}}{\partial \omega_2}\right)\\ &\left.\cdot\left(\frac{d(\delta\phi_1)}{dt} - j\frac{d(\delta v_1)}{dt}\right)\right\} = 0\end{aligned} \tag{14}$$

where

$$\delta v_1 = \frac{\delta V_1}{V_{10}}, \frac{\partial p_{ko}}{\partial v_l} = V_{lo} \cdot \frac{\partial p_{ko}}{\partial V_l} \quad \text{and} \quad \frac{\partial q_{ko}}{\partial v_l} = V_{lo} \cdot \frac{\partial q_{ko}}{\partial V_l},$$

for $l = 1, 2$.

Because the operating point is independent of the choice of reference phase, (14) is independent of $\delta\phi_1$. Thus

$$\frac{\partial p_{ko}}{\partial \phi_1} + \frac{2\partial p_{ko}}{\partial \phi_2} = 0 \tag{15}$$

and

$$\frac{\partial q_{ko}}{\partial \phi_1} + \frac{2\partial q_{ko}}{\partial \phi_2} = 0. \tag{16}$$

Defining

$$\frac{\partial p_{ko}}{\partial \omega_1} + \frac{2\partial p_{ko}}{\partial \omega_2} \triangleq \frac{dp_{ko}}{d\omega_1} \tag{17}$$

and

$$\frac{\partial q_{ko}}{\partial \omega_1} + \frac{2\partial q_{ko}}{\partial \omega_2} \triangleq \frac{dq_{ko}}{d\omega_1} \tag{18}$$

and equating real and imaginary parts of (14) to zero, we obtain

$$\left(\frac{\partial p_{ko}}{\partial v_1} + \frac{2\partial p_{ko}}{\partial v_2}\right)\delta v_1 + \frac{dq_{ko}}{d\omega_1}\delta\dot{v}_1 + \frac{dp_{ko}}{d\omega_1}\delta\dot{\phi}_1 + \frac{\partial p_{ko}}{\partial v_2}b + \frac{\partial p_{ko}}{\partial \phi_2}a = 0 \tag{19}$$

$$\left(\frac{\partial q_{ko}}{\partial v_1} + \frac{2\partial q_{ko}}{\partial v_2}\right)\delta v_1 - \frac{dp_{ko}}{d\omega_1}\delta\dot{v}_1 + \frac{dq_{ko}}{d\omega_1}\delta\dot{\phi}_1 + \frac{\partial q_{ko}}{\partial v_2}b + \frac{\partial q_{ko}}{\partial \phi_2}a = 0 \tag{20}$$

where the dot denotes the time derivative.

These equations may be solved by eliminating a, b, and $\delta\dot{\phi}$ to yield an equation of the form

$$\delta\dot{v} + Z\delta v = 0 \tag{21}$$

where Z is a scalar quantity of the form

$$Z = \frac{\left(\frac{\gamma}{\alpha} - \frac{\eta}{\epsilon}\right)}{\left(\frac{\beta}{\alpha} - \frac{\mu}{\epsilon}\right)}. \tag{22}$$

Expressions for α, β, γ, ϵ, μ, and η are readily calculated using the general formulas given in the following section. The solution to (21) has the form $\delta v = Ae^{-Zt}$ where A is determined by the initial conditions of δv. Stability of the oscillation state requires δv to decay, and thus requires $Z > 0$.

III. General Multifrequency Oscillator Stability

The approach set out in the preceding section is readily generalized to the case of an oscillator in which there are N frequency components in the output. We take the time-varying voltage component across the device to be

$$V(t) = \sum_{n=1}^{N} V_n \cos(\omega_n t + \phi_n) \tag{23}$$

and proceed as in the previous section.

Applying the Kurokawa condition at each frequency, we obtain

$$p_k + jq_k = 0, \qquad \text{for } k = 1, 2, \cdots, N. \tag{24}$$

As before, let the operating point be perturbed by a small amount such that

$$p_k \to p_{ko} + dp_k \tag{25}$$

$$q_k \to q_{ko} + dq_k \tag{26}$$

$$V_k \to V_{ko} + \delta V_k \tag{27}$$

$$\phi_k \to \phi_{ko} + \delta\phi_k \tag{28}$$

$$\omega_k \to \omega_{ko} + \delta\omega_k. \tag{29}$$

Define vector quantities $\boldsymbol{\delta v}$, $\boldsymbol{\delta\phi}$, and $\boldsymbol{\delta\omega}$ by

$$\boldsymbol{\delta v} = \left[\frac{\delta V_1}{V_{10}} \frac{\delta V_2}{V_{20}} \cdots \frac{\delta V_N}{V_{N0}}\right]^T \tag{30}$$

$$\boldsymbol{\delta\phi} = [\delta\phi_1\delta\phi_2 \cdots \delta\phi_N]^T \tag{31}$$

$$\boldsymbol{\delta\omega} = [\delta\omega_1\delta\omega_2 \cdots \delta\omega_N]^T \tag{32}$$

where the T denotes the transposed vector. We can then expand (24) in a Taylor series about the operating point and express the result in matrix form to obtain

$$Q\boldsymbol{\delta v} + P\boldsymbol{\delta\phi} + W\boldsymbol{\delta\omega} = 0 \tag{33}$$

where Q, P, and W are square matrices whose elements are defined by

$$Q_{kl} = V_{lo}\left(\frac{\partial p_{ko}}{\partial V_l} + j\frac{\partial q_{ko}}{\partial V_l}\right) \tag{34}$$

$$P_{kl} = \frac{\partial p_{ko}}{\partial \phi_l} + j\frac{\partial q_{ko}}{\partial \phi_l} \tag{35}$$

$$W_{kl} = \frac{\partial p_{ko}}{\partial \omega_l} + j\frac{\partial q_{ko}}{\partial \omega_l} \tag{36}$$

for $k=1,2,\cdots,N$ and $l=1,2,\cdots,N$.

As before, after Kurokawa [1], we have the relation

$$\boldsymbol{\delta\omega} = \boldsymbol{\delta\dot{\phi}} - j\boldsymbol{\delta\dot{v}}, \tag{37}$$

where the dot denotes the derivative with respect to time.

Let there be M independent oscillation frequencies ω_m, for $m=1,2,\cdots,M$ and $M \leqslant N$. The ω_m form a subset of the ω_n in (23). For purely harmonic oscillations, there is only one independent frequency, the fundamental. However, there also may be other independent frequencies present: for example, those arising from a low-frequency oscillation or due to other spurious circuit resonances. The frequencies present in the system may be related to the independent frequencies by a matrix L (of dimensions $N \times M$) such that

$$\boldsymbol{\omega} = L\boldsymbol{\omega}_i \tag{38}$$

where $\boldsymbol{\omega}_i$ is a vector of length M with elements ω_m.

Let the perturbations in the independent frequencies be given by $\boldsymbol{\delta\omega}_i$, and the corresponding voltage and phase perturbations by $\boldsymbol{\delta v}_i$ and $\boldsymbol{\delta\phi}_i$, respectively. We then have

$$\boldsymbol{\delta\omega} = L\boldsymbol{\delta\omega}_i. \tag{39}$$

Substituting from (37), we find

$$\boldsymbol{\delta\omega} = \boldsymbol{\delta\dot{\phi}} - j\boldsymbol{\delta\dot{v}} = L(\boldsymbol{\delta\dot{\phi}}_i - j\boldsymbol{\delta\dot{v}}_i). \tag{40}$$

Integration of (40) gives

$$\boldsymbol{\delta\phi} - j\boldsymbol{\delta v} = L(\boldsymbol{\delta\phi}_i - j\boldsymbol{\delta v}_i) + T(\boldsymbol{a} + j\boldsymbol{b}) \tag{41}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are constant vectors of length $N-M$. T is an $N \times (N-M)$ matrix, which is obtained from an $N \times N$ unit matrix by deleting the M *columns* corresponding to the ω_m. These are the same columns as those of the matrix L which have $L_{kk}=1$ for $k=1,2,\cdots,N$, i.e., the columns with unity on the diagonal of L. This relationship arises because there is one complex constant for each dependent frequency.

Separating (41) into real and imaginary parts, we have

$$\boldsymbol{\delta\phi} = L\boldsymbol{\delta\phi}_i + T\boldsymbol{d} \tag{42}$$

$$\boldsymbol{\delta v} = L\boldsymbol{\delta v}_i + T\boldsymbol{c} \tag{43}$$

where, for convenience, we have put

$$\boldsymbol{d} = \boldsymbol{a}$$

and

$$\boldsymbol{c} = -\boldsymbol{b}.$$

Substituting for $\boldsymbol{\delta\phi}$ and $\boldsymbol{\delta v}$ in (33) and using (40), we obtain

$$QL\boldsymbol{\delta v}_i + QT\boldsymbol{c} + PL\boldsymbol{\delta\phi}_i + PT\boldsymbol{d} + WL(\boldsymbol{\delta\dot{\phi}}_i - j\boldsymbol{\delta\dot{v}}_i) = 0. \tag{44}$$

Since there is an arbitrary phase reference associated with each independent frequency, then PL must be a zero matrix. Thus, we may write (41) as

$$A\boldsymbol{\delta\dot{\phi}}_i - jA\boldsymbol{\delta\dot{v}}_i + B\boldsymbol{\delta v}_i + C\boldsymbol{c} + D\boldsymbol{d} = 0 \tag{45}$$

where

$$A = WL$$
$$B = QL$$
$$C = QT$$
$$D = PT.$$

Equating real and imaginary parts to zero, we have

$$A_p\boldsymbol{\delta\dot{\phi}}_i + A_q\boldsymbol{\delta\dot{v}}_i + B_p\boldsymbol{\delta v}_i + C_p\boldsymbol{c} + D_p\boldsymbol{d} = 0 \tag{46}$$

$$A_q\boldsymbol{\delta\dot{\phi}}_i - A_p\boldsymbol{\delta\dot{v}}_i + B_q\boldsymbol{\delta v}_i + C_q\boldsymbol{c} + D_q\boldsymbol{d} = 0 \tag{47}$$

where $A = A_p + jA_q$, $B = B_p + jB_q$, $C = C_p + jC_q$, and $D = D_p + jD_q$. Thus, we have $2N$ equations in $2N$ unknowns, i.e., M phase angles $\delta\phi_{i_m}$, M voltages δv_{i_m}, and $2(N-M)$ constants c_k and d_k. The method of solution is to solve the $2(N-M)$ equations corresponding to the dependent frequencies for $\boldsymbol{c}$ and $\boldsymbol{d}$ in terms of $\boldsymbol{\delta\phi}_i$ and $\boldsymbol{\delta v}_i$, and then to substitute these values in the remaining $2M$ equations to solve for $\boldsymbol{\delta\phi}_i$ and $\boldsymbol{\delta v}_i$. As stability depends only on $\boldsymbol{\delta v}_i$, we need only to solve for $\boldsymbol{\delta v}_i$ in the form

$$\boldsymbol{\delta\dot{v}}_i + Z\boldsymbol{\delta v}_i = 0 \tag{48}$$

where Z is an $M \times M$ matrix. The system will be stable if and only if Z has eigenvalues with positive real parts.

The solution proceeds as follows: We separate the system of equations in (46) and (47) by multiplying through by matrices U and S. Here, $U = T^T$ and S is the unit matrix with the rows corresponding to the *dependent* frequencies deleted. Thus, the columns of S combined with the columns of U constitute a unit matrix. Now let

$$A_{pu} = UA_p, \qquad B_{pu} = UB_p, \text{ etc.} \tag{49}$$

Similarly let

$$A_{ps} = SA_p, \qquad B_{ps} = SB_p, \text{ etc.} \tag{50}$$

Equations (46) and (47) then may be written as

$$A_{ps}\boldsymbol{\delta\dot{\phi}}_i + A_{qs}\boldsymbol{\delta\dot{v}}_i + B_{ps}\boldsymbol{\delta v}_i + C_{ps}\boldsymbol{c} + D_{ps}\boldsymbol{d} = 0 \tag{51}$$

$$A_{qs}\boldsymbol{\delta\dot{\phi}}_i - A_{ps}\boldsymbol{\delta\dot{v}}_i + B_{qs}\boldsymbol{\delta v}_i + C_{qs}\boldsymbol{c} + D_{qs}\boldsymbol{d} = 0 \tag{52}$$

$$A_{pu}\boldsymbol{\delta\dot{\phi}}_i + A_{qu}\boldsymbol{\delta\dot{v}}_i + B_{pu}\boldsymbol{\delta v}_i + C_{pu}\boldsymbol{c} + D_{pu}\boldsymbol{d} = 0 \tag{53}$$

$$A_{qu}\boldsymbol{\delta\dot{\phi}}_i + A_{pu}\boldsymbol{\delta\dot{v}}_i + B_{qu}\boldsymbol{\delta v}_i + C_{qu}\boldsymbol{c} + D_{qu}\boldsymbol{d} = 0. \tag{54}$$

From (53) and (54)

$$\boldsymbol{c} = -X_{DC}^{-1}\{X_{DA}\boldsymbol{\delta\dot{\phi}}_i + Y_{DA}\boldsymbol{\delta\dot{v}}_i + X_{DB}\boldsymbol{\delta v}_i\} \tag{55}$$

$$\boldsymbol{d} = -X_{CD}^{-1}\{X_{CA}\boldsymbol{\delta\dot{\phi}}_i + Y_{CA}\boldsymbol{\delta\dot{v}}_i + X_{CB}\boldsymbol{\delta v}_i\} \tag{56}$$

where

$$X_{DC} = D_{pu}^{-1}C_{pu} - D_{qu}^{-1}C_{qu}$$

$$Y_{DA} = D_{pu}^{-1}A_{qu} + D_{qu}^{-1}A_{pu}$$

and so on.

Substituting these values into (51) and (52), we obtain

$$E_p\boldsymbol{\delta\dot{\phi}}_i + F_p\boldsymbol{\delta\dot{v}}_i + G_p\boldsymbol{\delta v}_i = 0 \tag{57}$$

$$E_q\boldsymbol{\delta\dot{\phi}}_i + F_q\boldsymbol{\delta\dot{v}}_i + G_q\boldsymbol{\delta v}_i = 0 \tag{58}$$

where

$$E_p = A_{ps} - C_{ps} X_{DC}^{-1} X_{DA} - D_{ps} X_{CD}^{-1} X_{CA}$$

$$F_p = A_{qs} - C_{ps} X_{DC}^{-1} Y_{DA} - D_{ps} X_{CD}^{-1} Y_{CA}$$

$$G_p = B_{ps} - C_{ps} X_{DC}^{-1} X_{DB} - D_{ps} X_{CD}^{-1} X_{CB}$$

$$E_q = A_{qs} - C_{qs} X_{DC}^{-1} X_{DA} - D_{qs} X_{CD}^{-1} X_{CA}$$

$$F_q = - A_{ps} - C_{qs} X_{DC}^{-1} Y_{DA} - D_{qs} X_{CD}^{-1} Y_{CA}$$

and

$$G_q = B_{qs} - C_{qs} X_{DC}^{-1} X_{DB} - D_{qs} X_{CD}^{-1} X_{CB}.$$

Thus, eliminating $\delta\phi_i$ from (57) and (58), we obtain

$$\delta\dot{v}_i + Z\delta v_i = 0 \tag{59}$$

where

$$Z = \left(E_p^{-1}F_p - E_q^{-1}F_q\right)^{-1} \cdot \left(E_p^{-1}G_p - E_q^{-1}G_q\right).$$

The system is stable if and only if Z has eigenvalues with positive real parts [5].

IV. Discussion

In the discussion that follows, we examine the application of the stability matrix Z, given by (59), to three cases of particular interest: a) a single frequency, b) a fundamental and second-harmonic, and c) a three-frequency parametric system.

A. Single-Frequency Case

If only one frequency is considered, (59) should yield the familiar Kurokawa stability condition. In this case, $N = M = 1$ and there are no constants c and d. Also $L = 1$, $S = 1$, and $U = 0$. Thus, from (57) and (58)

$$E_p = A_p = \frac{\partial p}{\partial \omega}, \quad F_p = A_q = \frac{\partial q}{\partial \omega}, \quad G_p = B_p = \frac{V\partial p}{\partial V}$$

$$E_q = A_q = \frac{\partial q}{\partial \omega}, \quad F_q = -A_p = -\frac{\partial p}{\partial \omega}, \quad G_q = B_q = \frac{V\partial q}{\partial V}.$$

Thus

$$Z = \left(\frac{\frac{V\partial p}{\partial V}}{\frac{\partial p}{\partial \omega}} - \frac{\frac{V\partial q}{\partial V}}{\frac{\partial q}{\partial \omega}} \right) \Bigg/ \left(\frac{\frac{\partial q}{\partial \omega}}{\frac{\partial p}{\partial \omega}} + \frac{\frac{\partial p}{\partial \omega}}{\frac{\partial q}{\partial \omega}} \right)$$

i.e.,

$$Z = \frac{V\left(\frac{\partial p}{\partial V} \cdot \frac{\partial q}{\partial \omega} - \frac{\partial q}{\partial V} \cdot \frac{\partial p}{\partial \omega}\right)}{\left(\frac{\partial p}{\partial \omega}\right)^2 + \left(\frac{\partial q}{\partial \omega}\right)^2}. \tag{60}$$

Thus, $Z > 0$ requires

$$\left(\frac{\partial p}{\partial V} \cdot \frac{\partial q}{\partial \omega} - \frac{\partial q}{\partial V} \cdot \frac{\partial p}{\partial \omega}\right) > 0$$

which, by recalling the definition of p and q from (3), can be recognized as the usual stability criteria for the single-frequency oscillator [1], [6].

It is worth noting that, although the phase reference is arbitrary, we cannot set $\delta\phi$ to zero, because the phase suffers perturbations about the arbitrary reference phase. The rate of change of this phase deviation with respect to time (cf., frequency and phase modulation) is then equal to the real part of the frequency deviation. However, $\partial p/\partial \phi$ and $\partial q/\partial \phi$ are identically zero as the device impedance is independent of the phase perturbations.

B. Two Harmonically Related Frequencies

For two harmonically related frequencies, Z is a scalar quantity of the form given by (22). Values for α, β, γ, ϵ, η, and γ may now be determined using the general expressions in (57) and (58). For a fundamental and second-harmonic

$$L = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad S = [1 \;\; 0], \text{ and } U = [0 \;\; 1]. \tag{61}$$

Thus, from (45), (49), and (50)

$$A_{ps} = \frac{\partial p_1}{\partial \omega_1} + \frac{2\partial p_1}{\partial \omega_2} \triangleq \frac{dp_1}{d\omega_1}, \quad A_{qs} = \frac{\partial q_1}{\partial \omega_1} + \frac{2\partial q_1}{\partial \omega_2} \triangleq \frac{dq_1}{d\omega_1} \tag{62}$$

$$B_{ps} = \frac{\partial p_1}{\partial v_1} + \frac{2\partial p_1}{\partial v_2}, \quad B_{qs} = \frac{\partial q_1}{\partial v_1} + \frac{2\partial q_1}{\partial v_2} \tag{63}$$

$$C_{ps} = \frac{\partial p_1}{\partial v_2}, \quad C_{qs} = \frac{\partial q_1}{\partial v_2} \tag{64}$$

$$D_{ps} = \frac{\partial p_1}{\partial \phi_2}, \quad D_{qs} = \frac{\partial q_1}{\partial \phi_2}. \tag{65}$$

Replacing s by u, p_1 by p_2, and q_1 by q_2 in (62)–(65), we obtain the values of A_{pu}, A_{qu}, B_{pu}, B_{pu}, C_{pu}, C_{qu}, D_{pu}, and D_{qu}. Thus, in (55) and (56)

$$X_{DC} = \left[\frac{\frac{\partial p_2}{\partial v_2}}{\frac{\partial p_2}{\partial \phi_2}} - \frac{\frac{\partial q_2}{\partial v_2}}{\frac{\partial q_2}{\partial \phi_2}} \right], \quad X_{CD} = \left[\frac{\frac{\partial p_2}{\partial \phi_2}}{\frac{\partial p_2}{\partial v_2}} - \frac{\frac{\partial q_2}{\partial \phi_2}}{\frac{\partial q_2}{\partial v_2}} \right] \tag{67}$$

$$X_{DA} = \left[\frac{\frac{dp_2}{d\omega_1}}{\frac{\partial p_2}{\partial \phi_2}} - \frac{\frac{dq_2}{d\omega_1}}{\frac{\partial q_2}{\partial \phi_2}} \right], \quad X_{CA} = \left[\frac{\frac{dp_2}{d\omega_1}}{\frac{\partial p_2}{\partial v_2}} - \frac{\frac{dq_2}{d\omega_1}}{\frac{\partial q_2}{\partial v_2}} \right] \tag{68}$$

$$Y_{DA} = \left[\frac{\frac{dq_2}{d\omega_1}}{\frac{\partial p_2}{\partial \phi_2}} + \frac{\frac{dp_2}{d\omega_1}}{\frac{\partial q_2}{\partial \phi_2}} \right], \quad Y_{CA} = \left[\frac{\frac{dq_2}{d\omega_1}}{\frac{\partial p_2}{\partial v_2}} + \frac{\frac{dp_2}{d\omega_1}}{\frac{\partial q_2}{\partial \phi_2}} \right] \tag{69}$$

$$X_{DB} = \left[\frac{\frac{\partial p_2}{\partial v_1} + \frac{2\partial p_2}{\partial v_2}}{\frac{\partial p_2}{\partial \phi_2}} - \frac{\frac{\partial q_2}{\partial v_1} + \frac{2\partial q_2}{\partial v_2}}{\frac{\partial q_2}{\partial \phi_2}} \right]$$

$$X_{CB} = \left[\frac{\frac{\partial p_2}{\partial v_1} + \frac{2\partial p_2}{\partial v_2}}{\frac{\partial p_2}{\partial v_2}} - \frac{\frac{\partial q_2}{\partial v_1} + \frac{2\partial q_2}{\partial v_2}}{\frac{\partial q_2}{\partial \phi_2}} \right]. \tag{70}$$

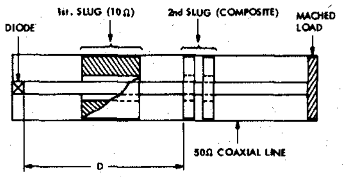

Fig. 2. Coaxial oscillator circuit for study of second-harmonic tuning.

The required parameters α, β, γ, ϵ, η, and μ may be determined from the expressions in (57) and (58) for E_p, F_p, G_p, E_q, F_q, and G_q, respectively. Taking α as a typical term and writing it in full, we obtain the expression

$$\alpha = \frac{dp_1}{d\omega_1} - \frac{\partial p_1}{\partial v_2}\left[\frac{\dfrac{dp_2}{d\omega_1}}{\dfrac{\partial p_2}{\partial \phi_2}} - \frac{\dfrac{dq_2}{d\omega_1}}{\dfrac{\partial q_2}{\partial \phi_2}}\right] \Bigg/ \left[\frac{\dfrac{\partial p_2}{\partial v_2}}{\dfrac{\partial p_2}{\partial \phi_2}} - \frac{\dfrac{\partial q_2}{\partial v_2}}{\dfrac{\partial q_2}{\partial \phi_2}}\right]$$
$$- \frac{\partial p_1}{\partial \phi_2}\left[\frac{\dfrac{dp_2}{d\omega_1}}{\dfrac{\partial p_2}{\partial v_2}} - \frac{\dfrac{dq_2}{d\omega_1}}{\dfrac{\partial q_2}{\partial v_2}}\right] \Bigg/ \left[\frac{\dfrac{\partial p_2}{\partial \phi_2}}{\dfrac{\partial p_2}{\partial v_2}} - \frac{\dfrac{\partial q_2}{\partial \phi_2}}{\dfrac{\partial q_2}{\partial v_2}}\right]. \quad (70)$$

From this expression, the effect of including the second harmonic in the stability analysis can be seen. If only the fundamental is considered, $\alpha = \partial p_1/\partial \omega_1$. The additional terms arising when the second harmonic is considered depend on changes in the fundamental impedance due to the presence of the harmonic as well as the changes in the second-harmonic impedance itself. Note that, in general, terms like $\partial p_1/\partial v_2$ and $\partial p_1/\partial \phi_2$, which appear as multiplying terms in (70), will be small and therefore, as would be expected, the overall influence of harmonic terms will be small. However, for example, a resonance at the second-harmonic frequency will result in a large $\partial q_2/\partial \omega_2$ (recall that q_2 is the *total* reactance including the external circuit reactance). Depending on the sign and magnitude of the other terms, this term could have either a stabilizing or destabilizing effect. This could be particularly important in the design of self-oscillating harmonic generators [7].

The influence of second-harmonic interactions on oscillator stability is investigated by considering some application examples. We consider the multiple slug-tuned coaxial circuit shown in Fig. 2, which permits independent tuning of the fundamental and second-harmonic at the design frequency. Similar circuits have been used previously in the study of second-harmonic effects in IMPATT circuits [8], [10].

The oscillator circuit was designed for operation at 13.5 GHz. The 10-Ω slug nearest to the diode is $\lambda/4$ long, where λ is the wavelength at 13.5 GHz and thus has no effect on the impedance at the second harmonic. The second slug is a composite slug formed of two $\lambda/6$ slugs with a fixed spacing of 0.035 λ, such that the electrical length of the two $\lambda/6$ slugs plus the gap is $\lambda/2$. This slug then has no effect on the fundamental impedance. At the second-harmonic frequency, the second slug presents a large impedance mismatch approaching that of a short circuit, preventing second-harmonic power from reaching the load. Thus, by keeping the position of the first slug fixed and moving the second (composite) slug, the impedance at the second harmonic may be varied relatively independently of the fundamental impedance. Because the design is frequency sensitive, the impedances are completely independent only at the design frequency. However, as long as the position of the first slug is fixed, the variations in oscillator frequency are small enough for the impedances to be considered independent.

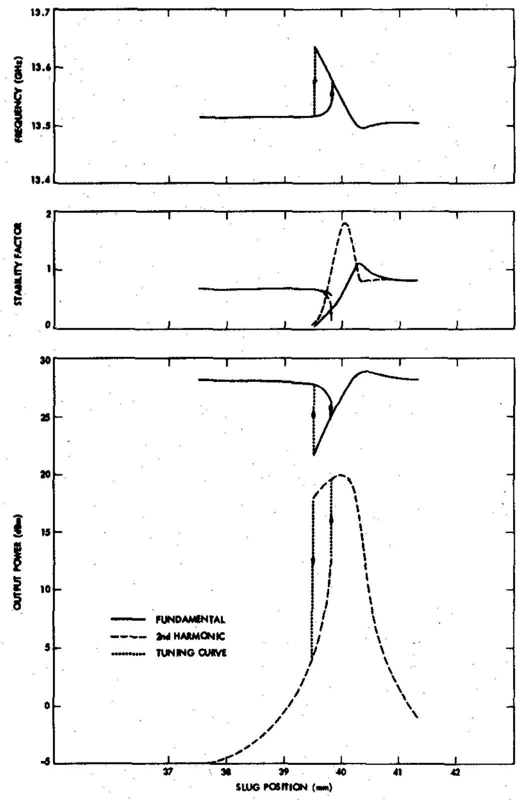

Fig. 3. Calculated frequency, stability factor, and output power as a function of second-harmonic slug position for a 13.5-GHz oscillator.

The oscillation frequency and output power were determined from a circuit model of the oscillator structure incorporating a nonlinear IMPATT diode data using the analysis method of Bates and Khan [9], [10]. The derivatives required to determine stability are calculated as part of the minimization technique used to find the operating point. The parameters of the IMPATT diode were derived from typical X-band silicon IMPATT diode data.

Fig. 3 shows oscillation frequency, output power, and stability factor Z as a function of the position of the composite slug, i.e., as a function of second-harmonic impedance. The stability factor was calculated using both the single-frequency stability criterion (due to Kurokawa) and the two-frequency expressions derived in this paper. Notice that, although no instability is indicated for either expression (Z is always positive), there is a significant difference in the value of Z when the second-harmonic output is large (less than 15 dB below the fundamental). Note also the hysteresis in the tuning characteristic associated with large second-harmonic output power.

Consider now an example in which the second harmonic does influence stability. The same basic circuit is used as

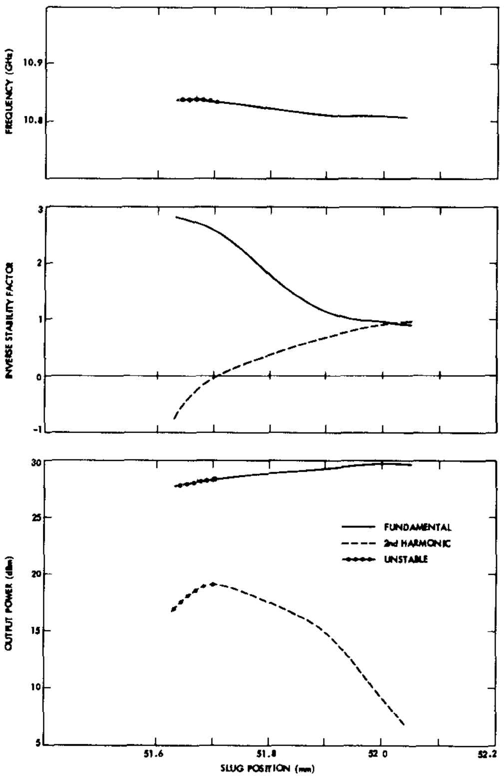

Fig. 4. Calculated frequency, inverse stability factor, and output power as a function of second-harmonic slug position for a 10.8-GHz oscillator.

previously, only the dimensions are modified so that the circuit oscillates at 10.8 GHz, and the independent tuning condition applies at this frequency. The calculated oscillation frequency, output power, and inverse stability factor are shown in Fig. 4 as a function of the position of the composite slug. The inverse of Z, rather than Z, is plotted because, in the second-harmonic case, Z has a pole. However, we are interested primarily in the sign of Z. Note that now the single-frequency stability criterion indicates stable operation everywhere, but the two-frequency criterion indicates unstable operation for slug positions less than 51.7 mm.

In an attempt to understand the source of the instability, we examined the terms of the stability expression and found that the stability factor changes sign at the point where

$$\frac{E_p}{E_q} = \frac{F_p}{F_q}.$$

Because E_p, E_q, F_p, and F_q are all complicated functions of derivatives of impedance with respect to amplitude, frequency, and phase, it is apparent that the source of the instability cannot be attributed to any particular term, but rather is due to many interacting derivative terms.

C. Three Parametrically Related Frequencies

Consider a three-frequency parametric system with two independent oscillation frequencies ω_1 and ω_2, and a third frequency $\omega_3 = \omega_1 - \omega_2$. Thus

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}.$$

This may arise in an oscillator with a fundamental frequency ω_1 and a low-frequency oscillation ω_2. This situation is generally undesirable and can be avoided by proper design procedures [11], [12]. However, it may be worthwhile to examine this case because of its application to self-oscillating frequency converters and because the resulting expressions are believed to relate closely to the response of the oscillator to internal and external noise sources [13] or to injected signals [1]. That is, as the oscillator nears an unstable condition, noise sources near the frequency at which the instability occurs become amplified and thus the oscillator output becomes noisy. The relevance of the stability expressions to oscillator noise performance warrants investigation, but is beyond the scope of this paper.

For a parametric system, $N = 3$ and $M = 2$. Thus

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}, \quad T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and

$$U = [0 \quad 0 \quad 1].$$

Using these matrices and following the solution method given, we may determine the 2×2 stability matrix Z. This matrix indicates stability if it has eigenvalues with positive real parts. This condition is satisfied if the determinant and the sum of the principal diagonal elements are both greater than zero [5], i.e., for stability

$$|Z| > 0$$

and

$$Z_{11} + Z_{22} > 0.$$

V. Experiment

Experimental verification of the stability expressions derived in this paper is a formidable task, made difficult by the impracticality of decisively identifying an unstable operating point or experimentally measuring the stability factor. Hysteresis in tuning characteristics, spurious oscillation, or abrupt changes in output power and frequency all result from unstable operating points, but circuit conditions other than instability can also cause these phenomena. However, because the stability analysis requires first the theoretical determination of the oscillator operating point, results are presented here to show that accurate theoretical determination of the oscillation state is possible and that, for those theoretically determined oscillation points verified experimentally, the stability analysis indicates stable operation.

The IMPATT oscillator used in the experiments is shown in Fig. 5. This circuit was chosen because it is known to be prone to spurious oscillations, frequency jumping, and noisy output as the position of a tuning slug or the diode bias current is varied. It is thus particularly suitable for studying instabilities. The coaxial structure can also be readily and accurately modeled, provided care is taken to account for the discontinuity capacitances associated with the diode mount and the tuning slugs. Details of the oscillator circuit, and the modeling and analysis of the

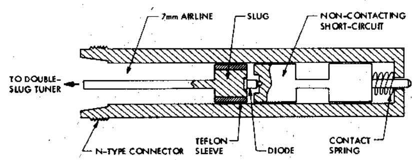

Fig. 5. Experimental coaxial oscillator circuit. A double-slug coaxial tuner was used for tuning.

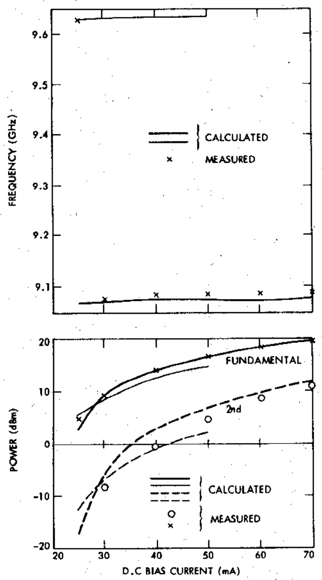

Fig. 6. Comparison of measured and calculated frequency and fundamental and second-harmonic output powers as a function of dc bias current. Two separate solutions of the oscillator equations are indicated.

IMPATT diode and circuit are given elsewhere [9], [10]. It should be emphasized that no RF measurements were necessary to determine the parameters of either the diode or the circuit and the only estimated parameter was the series resistance of the diode. The oscillator frequency output power and the diode and circuit impedances and their derivatives used in the stability calculations were all determined theoretically; it would be impractical to measure the derivatives experimentally.

Fig. 6 shows the measured and calculated fundamental and second-harmonic powers delivered to the load as a function of dc bias current, for a slug spacing of 105 mm. At the threshold current of 25 mA, the measured frequency of oscillation was 9.627 GHz. However, as the bias current was increased to 30 mA, the oscillation jumped to 9.080 GHz and then increased slowly with bias current to 9.092 GHz at 70 mA. This behavior is typical of multiple-tuned oscillator circuits. The output power at 25 mA was 4.2 dBm and increased smoothly to 19.4 dBm, despite the frequency jump. The total output power was measured with a power meter, while the second-harmonic output power was determined by measuring the relative power difference on a spectrum analyzer.

The theoretical results show good agreement with the experimental values, including the two distinct oscillation frequencies. However, the theory cannot predict at which of the two frequencies the circuit will oscillate, as this depends on transient behavior and the history of circuit adjustment. At 50 mA bias, the calculated value of the

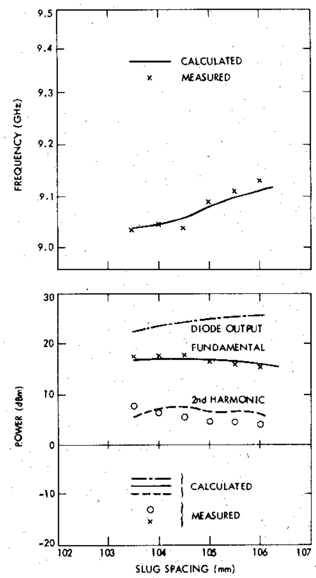

Fig. 7. Comparison of measured and calculated frequency and fundamental and second-harmonic output powers as a function of slug spacing. Also shown is the power produced at the diode terminals.

TABLE I
COMPARISON OF MEASURED AND CALCULATED FREQUENCY AND POWER FOR THE PARAMETRIC OSCILLATOR

	CALCULATED		MEASURED	
FREQ. NO.	FREQ. (GHz)	POWER (dBm)	FREQ. (GHz)	POWER (dBm)
fp	9.723	13.55	9.771	15.41
fo	1.450	-2.15	1.498	-11.09
f-1	8.273	1.43	8.273	5.41

stability factor Z was 0.22 ns^{-1} for the higher frequency mode and 0.072 ns^{-1} for the lower frequency mode, i.e., both modes are stable.

Fig. 7 shows the measured and calculated frequency and output power at a bias current of 50 mA as a function of the spacing between the slugs as the slug nearest the diode was moved. Also shown is the RF power produced in the diode, indicating a circuit loss between the diode and the load ranging from 5 to 10 dB. As before, good agreement exists between measured and calculated values. The stability analysis indicates stable operation for all the calculated values shown. However, although the experimental oscillator breaks into parametric oscillation for a slug spacing greater than 106 mm, no instability at this point is predicted by the analysis. This may be because other frequency components not included in the analysis become important under these conditions. In fact, at some slug positions, the output spectrum of the oscillator showed in excess of ten frequency components that were not harmonically related.

Finally, we consider a three-frequency parametric-oscillator, in which the three frequencies f_p, f_o, and f_{-1} satisfy the relation

$$f_{-1} = f_p - f_o.$$

Table I gives a comparison of the measured and calculated frequency and output power values. The calculated stability matrix was

$$Z = \begin{bmatrix} 0.098 & -0.390 \\ 0.131 & 0.620 \end{bmatrix}.$$

Thus, $|Z| = 0.112 > 0$ and $Z_{11} + Z_{22} = 0.718 > 0$, indicating a stable (parametric) system. The maximum difference between the measured and calculated frequency is 40 MHz, while the agreement in output power is excellent at frequencies f_p and f_{-1}, but is in error by about 9 dB at f_o. However, the output power is very sensitive to the real part of the impedance at this frequency.

VI. Conclusion

Expressions have been derived which permit determination of the stability with respect to small perturbations in the operating point of a negative-resistance oscillator with a number of arbitrarily related frequency components present in the output. Although the expressions are complicated and the measurement of the various terms impractical, the expressions are easily calculated from theoretical diode and circuit models with the aid of a computer. These expressions should therefore find particular application in the computer-aided design of solid-state oscillators and harmonic generators [14].

Acknowledgment

The authors thank M. B. Steer for providing the computer programs used for nonlinear analysis.

References

[1] K. Kurokawa, "Some basic characteristics of broadband negative resistance oscillator circuits," *Bell Syst. Tech. J.*, vol. 48, pp. 1937–1955, July 1969.

[2] C. A. Brackett, "Characterization of second-harmonic effects in IMPATT diodes," *Bell Syst. Tech. J.*, vol., 49, pp. 1777–1810, Oct. 1970.

[3] K. W. Foulds and J. L. Sebastian, "Characteristics of negative-resistance nonsinusoidal oscillators," *IEEE Trans. Electron Devices*, vol. ED-25, pp. 646–655, June 1978.

[4] L. Gustafsson, G. H. B. Hansson, and K. I. Lundström, "On the use of describing functions in the study of nonlinear active microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 402–409, June 1972.

[5] S. Barnett, *Introduction to Mathematical Control Theory*. London: Oxford University Press, 1975, p. 150.

[6] G. H. B. Hansson and K. I. Lundström, "Stability criteria for phase-locked oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 641–645, Oct., 1972.

[7] E. Hagihara, M. Akaike, and K. Yamamoto, "Broad-band bias-current-tuned IMPATT oscillator for 100–200 GHz," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1927–1933, Nov. 1982.

[8] R. A. Giblin, K. G. Hambleton, and C. A. Tearle, "Octave tuning and the effect of second-harmonic loading of avalanche diode oscillators," *Electron. Lett.*, vol. 5, pp. 361–363, Aug. 7, 1969.

[9] B. D. Bates and P. J. Khan, "Analysis of waveguide IMPATT oscillator circuits," in *1981 IEEE/MTT-S Int. Microwave Symp. Dig.*, June 1981, pp. 232–234.

[10] B. D. Bates, "Nonlinear analysis of IMPATT oscillator circuits," Ph.D. dissertation, University of Queensland, Australia, Mar. 1982.

[11] C. A. Brackett, "The elimination of tuning induced burnout and bias-circuit oscillations in IMPATT oscillators," *Bell Syst. Tech. J.*, vol. 52, pp. 271–306, Mar. 1973.

[12] Y. Hirachi, T. Nakagami, Y. Toyama, and Y. Fukukawa, "High-power 50 GHz double-drift-region IMPATT oscillators with improved bias circuits for eliminating low-frequency instabilities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 731–737, Nov. 1976.

[13] I. Eddison and M. J. Howes, "Circuit aspects of the noise performance of microwave oscillator modules," *Inst. Elec. Eng. J. Microwaves, Optics and Acoustics*, vol. 1, pp. 103–109, Apr. 1977.

[14] V. Rizzoli, A. Lipparini, and E. Marazzi, "A general-purpose program for nonlinear microwave circuit design," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 762–769, Sept. 1983.

Bevan D. Bates (S'76–M'81) was born in Bundaberg, Australia, on February 29, 1956. He received the B.E., M.Eng.Sc., and Ph.D. degrees in electrical engineering, all from the University of Queensland, Australia, in 1976, 1979, and 1982, respectively.

From 1982 to 1983, he was a National Research Council Resident Research Associate at the Jet Propulsion Laboratory, Pasadena, CA, engaged in the design of millimeter-wave oscillators. He is currently Lecturer in Electrical Engineering at the University of Melbourne, Australia. His research interests include computer methods for the design and analysis of microwave and millimeter-wave circuits and the modeling of semiconductor devices.

Peter J. Khan (M'61–SM'79) was born in Bowral, Australia, on November 12, 1936. He received the B.Sc. degree in mathematics and physics, and the B.E. and Ph.D. degrees in electrical engineering, all from the University of Sydney, Australia, in 1957, 1959, and 1963, respectively.

From 1953 to 1959, he was employed at the Weapons Research Establishment at Salisbury, South Australia, carrying out research and development in electronic circuits. After completion of his doctoral studies in parametric amplification, he came to the University of Michigan, Ann Arbor, MI, in 1963, on a Fulbright Postdoctoral Fellowship. He remained there until 1976, as Assistant Professor and Associate Professor of Electrical Engineering. In 1976, he returned to Australia where he was Reader in Electrical Engineering at the University of Queensland. In late 1983, he took up an appointment at the Bahá'í World Centre, Haifa, Israel. His research interests include microwave solid-state circuit design, as well as fabrication and analysis of propagating structures at millimeter-wave and optical frequencies.